FOR APPROVAL **PUBLIC** **OPEN SESSION**

TO:	UTM Academic Affairs Committee
SPONSOR: **CONTACT INFO:**	Tracey Bowen, Vice-Dean, Teaching & Learning vdteachlearn.utm@utoronto.ca
PRESENTER: **CONTACT INFO:**	Marc Dryer, Associate Dean, Academic Programs marc.dryer@utoronto.ca
DATE:	January 4, 2024 for January 11, 2024
AGENDA ITEM:	4

ITEM IDENTIFICATION:

Major Modification: New Streams within an Existing Undergraduate Program (Major in Mathematical Sciences), UTM.

JURISDICTIONAL INFORMATION:

Under section 5.6 of its terms of reference, the Academic Affairs Committee is responsible for major and minor modifications to existing degree programs.

GOVERNANCE PATH:

1. **UTM Academic Affairs Committee [For Approval] (January 11, 2024)**

PREVIOUS ACTION TAKEN:

None.

HIGHLIGHTS:

This Major Modification to an academic program proposes the introduction of two streams into the existing, undifferentiated Mathematical Sciences Major (ERMAJ2511) offered by the Department of Mathematical and Computational Sciences. The parent program will be maintained, but it will house two distinct POSts: "**Mathematical Sciences – Major: Mathematics**", and "**Mathematical Sciences – Major: Applied Mathematics**". These two streams share the majority of their declared learning outcomes, with some unique to each stream.

Stream 1 (Major in Mathematical Sciences: Mathematics) will be the same as the current Major in Mathematical Sciences program in terms of its enrolment requirements, completion requirements, and learning outcomes. This stream would continue to be aimed at students with interest in mathematics or mathematics education and will continue to introduce the students to a broad range of mathematical subjects.

Stream 2 (Major in Mathematical Sciences: Applied Mathematics) is new. This will be a stream for students who would like to complete a mathematics major program, but with a focus on applications of mathematics to other fields. This focus will involve applying computational and statistical tools and skills to both study and create mathematical models of concrete, real-world phenomena. It will be well suited for students interested in completing a double Major of Applied Mathematics and Statistics or combining an Applied Mathematics Major with a Major in the Sciences. This stream will focus on mathematical and statistical subjects which are important for applications. Courses listed in the completion requirements for this new stream will draw upon examples from multiple disciplines which have a common mathematical core. Students combining this program with programs from a variety of other disciplines will therefore benefit from the same mathematics courses and will see a range of applications from their field and other fields.

Stream 2 will share the same enrolment requirements as Stream 1 but will have different program completion requirements (with some required courses in common).

FINANCIAL IMPLICATIONS:

There are no net implications for the campus' operating budget.

RECOMMENDATION:

Be It Resolved,

THAT the introduction of two new streams within the existing Mathematical Sciences Major (ERMAJ2511) offered by the Department of Mathematical and Computational Sciences, as detailed in the proposal dated December 19, 2023, be approved, effective September 1, 2024.

DOCUMENTATION PROVIDED:

Proposal for a Major Modification: New Streams within an Existing Undergraduate Program (Major in Mathematical Sciences), UTM

University of Toronto
Major Modification Proposal:

New Stream within an Existing Undergraduate Program

This template should be used to bring forward all proposals for new streams in existing undergraduate programs for governance approval under the University of Toronto's Quality Assurance Process.

Program name and degree: please specify specialist or major and clarify throughout whether streams are in both or one.	Major in Mathematical Sciences (HBSc) ERMAJ2511
Existing streams, if any:	N/A
Proposed new stream(s): specify whether this is in the specialist or major or both.	The existing, undifferentiated Major in Mathematical Sciences will become a program with two streams: 1. Mathematical Sciences Major: Mathematics will reflect the requirements and learning outcomes of the original program 2. Mathematical Sciences Major: Applied Mathematics is new
Faculty/academic division:	University of Toronto Mississauga (UTM); Department of Mathematical & Computational Sciences
Faculty/academic division contact:	Nick Rule Vice-Principal, Academic & Dean, UTM vpdean.utm@utoronto.ca
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Proponent:	Ilia Binder Chair, Department of Mathematical & Computational Sciences, UTM Ilia.binder@utoronto.ca
Version date: please change as you edit this proposal.	December 19, 2023

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1. Summary

- Please provide a brief summary or overview of how the proposed stream(s) relates to the existing program and any existing streams.

The Mathematical Sciences Major is housed in the Department of Mathematical and Computational Sciences at the University of Toronto Mississauga. Other programs supported by this unit include the Mathematical Sciences Specialist and Mathematical Sciences Minor.

The Mathematical Sciences Major is currently an undifferentiated program that leads to the Honours Bachelor of Science. With the approval of this proposal, the Major will become a program with two streams:

- Stream 1 will be called "Mathematical Sciences Major: Mathematics. It will consist of the same enrolment and program completion requirements, and the same program learning outcomes as the existing Major. This stream will continue to be aimed at, and appeal to, students with interest in mathematics or mathematics education and will continue to introduce the students to a broad range of mathematical subjects.
- Stream 2, which will be called "Mathematical Sciences Major: Applied Mathematics is new; however, it will have the same enrolment requirements as Stream 1, will share a common core of 4.0 FCE of course completion requirements with Stream 1, and have aligned program learning outcomes with Stream 1. This stream is intended for students who would like to complete a mathematics Major program, but with a focus on applications of mathematics to other fields. This focus will involve applying computational and statistical tools and skills to both study and create mathematical models of concrete, real-world phenomena, from biology to finance to robotics. It will be well suited for students interested in completing a double Major of Mathematical Sciences Major: Applied Mathematics and Statistics or combining a Mathematical Sciences Major: Applied Mathematics with a Major in the Sciences. Stream 2 will focus on mathematical and statistical subjects which are important for applications. Courses listed in the completion requirements for this new stream will draw upon examples from multiple disciplines which have a common mathematical core. Students combining Stream 2 with programs from a variety of other disciplines will therefore benefit from the same mathematics courses and will see a range of applications from their field and other fields.

Note: the naming of the streams ("Mathematics" and "Applied Mathematics") is standard and consistent with the established conventions of the field of Mathematical Sciences. This is what other institutions use (see some examples listed below in section 3). University program ratings (such as the US News and World report cited below in section 3) use this naming as well. Its meaning will be familiar to prospective employers, as well as students interested in Mathematical Sciences, and will appropriately convey the focus of each stream.

2. Effective Date

September 1, 2024

3. Academic Rationale

- What are the academic reasons for the change, the relationship with existing streams and how does the new stream fit with the unit's and division's academic plans?

The current Mathematical Sciences Major program prepares students to continue into an MSc or PhD program in Mathematics, or to pursue a broad range of other careers, including mathematics education. It provides a high-quality introduction to a wide range of topics in mathematics, ensuring that students are able to engage with advanced mathematical topics and proof techniques later in their careers. This Major can be combined with the more theoretical aspects of cognate disciplines, such as, theoretical Computer Science. It is an excellent quality program with scaffolded and well-structured course offerings and is on par with similar programs in North America at peer institutions.

However, while the strength of the current Mathematical Sciences Major is that it is broad and flexible, it doesn't have a strong emphasis on mathematical applications. This can be a weakness for students who wish to pursue a career focused on or requiring a heavy exposure to applications. There are already many existing courses that have a focus on application, and providing an offering that links them into a coherent and scaffolded program of study will be highly beneficial to students. Examples of careers that rely on this type of approach to mathematics, include Actuarial Science, Mathematical Finance, Mathematical Biology/Medicine/Epidemiology, Mathematical Linguistics, Mathematical Genetics, Mathematical Imaging, Big Data Analysis, Robotics, and many others. Some students in our current Mathematical Sciences Major do enter these fields, and they would be better served by a narrower focus on the mathematical tools needed in applications.

It is important to note, that while applications vary broadly, there is a core mathematical foundation for them. For instance, the study of Nonlinear Dynamics is equally relevant to Mathematical Biology, Epidemiology, Robotics, and Genetics. Aspects of the theory of Partial Differential Equations (PDEs) are used in all of the above, and also form the foundations for Mathematical Finance and Mathematical Imaging. And all of these applications rely heavily on Probability and Statistics as well as on Numerical Analysis. In other words, a student who studies Nonlinear Dynamics, PDEs, Statistics and Numerical Analysis will have the tools they need to succeed in a wide range of fields in applied mathematics.

While students in the current Mathematical Sciences Major *are able to* select courses that would cover these topics to an extent, the broadness of the program leaves students with less exposure to the relevant tools than what would be ideal to succeed in the above listed careers. On the other hand, the broadness of the existing Mathematical Sciences Major program and its more theoretical emphasis, are needed to ensure that graduates are prepared for continued studies in Mathematics or mathematics education.

The conclusion, which has been by now reached by most of our peer institutions, is that in order to direct the students on the path which is most relevant to their careers, more than one stream is needed. In fact, many our peer institutions already have at least one version of an applied mathematics stream, and often multiple versions with distinct areas of application. We name only a few examples below. This list could easily be made excessively long; it might be simpler to list the schools which do not have one.

- MIT Applied Mathematics stream (concentration) within the Math Major. Considered the best Applied Mathematics program in North America (USNews & World Report[1])
- McGill: Honours Applied Mathematics
- University of Waterloo: 14 streams in the Math major of which 10 are applied. Their math program description website[2] opens with: "Every company and industry needs logical thinkers like mathematicians and computer scientists to solve their everyday problems."
- University of Michigan: The Mathematical Sciences Program which "is designed to provide broad training in basic mathematics together with some specialization in an area of application of mathematics."[3] There are eight applied streams in the program.
- UCLA: Majors in: Applied Mathematics, Financial Actuarial Mathematics, Mathematics/Applied Science

We note that in the examples listed above, the larger academic units may have several applied streams within their applied programs (the University of Waterloo stands out in this regard) or additional, more narrow specialisations within the Applied Mathematics stream. Our situation is similar to smaller programs (such as McGill) with a single Applied Mathematics program. Larger departments (such as that found at MIT) often also offer a single Applied Mathematics program.

At present, many students with an interest in Mathematical applications at UTM have been electing to enrol into the Mathematical Sciences Minor. Every year we enrol a large cohort of students into the Minor, and currently we have about 1700 active students. In fact, there are far more students in the Mathematical Sciences Minor than in the Mathematical Sciences Major and Mathematical Sciences Specialist programs, combined. Many students currently in the Mathematical Sciences Minor or who are considering it as a means to develop understanding and skill that could be applied to other disciplines may be interested in the

[1] https://www.usnews.com/best-graduate-schools/top-science-schools/applied-mathematics-rankings
[2] https://uwaterloo.ca/math/future-undergraduates/programs
[3] https://lsa.umich.edu/math/undergraduates/major-and-minor-programs.html

new Stream 2. This is confirmed by the results of the student survey we present in the following section where we talk about the students' interest. While this may result in students moving from one program to another, the overall outcome is an enhancement in the quality of education they receive through a program deliberately designed to meet their needs.

Importantly, the proposed Stream 2 will be a better fit not only to careers in applied sciences or industry, but also to many MSc or PhD programs and research focus areas available in them. To take a few examples offered at the University of Toronto: the Master of Mathematical Finance[4], the Rotman PhD program (e.g. focusing on Optimization and Variational Methods[5]), the PhD in Industrial Engineering (e.g. focusing on Operations Research[6]), and the PhD program in Computer Science (e.g. focusing on Numerical Analysis[7]) or the Master of Applied Computing with a concentration in Applied Mathematics[8].

Of particular interest when considering a new offering in applied mathematics, the Department of Mathematical and Computational Sciences at UTM boasts some of the best faculty in Nonlinear Analysis/Dynamics in the world. Strength in this area is illustrated in the recently established EDU-C "Centre for Nonlinear Analysis and Modeling" at UTM. Non-linear Analysis/Dynamics is a central element in any applied mathematics program. Students in the new stream will benefit directly from the research-informed courses offered by these faculty.

In view of all of the above, the introduction of Stream 2 is seen as a key part of the future development and growth strategy of the MCS Department. Our latest (2022-23) self-study prepared for the upcoming UTQAP review states:

> "One effective strategy for improving the overall quality of the student experience would be the planned introduction of Robotics and **Applied Mathematics programs**. By incorporating these specialised programs into the curriculum, students would have the opportunity to develop key skills and knowledge in these highly relevant and rapidly advancing fields."

It further notes:

> "Perhaps **our most important opportunity right now is the opportunity to introduce a new Major Program in Applied Mathematics**. Most other comparable institutions across North America already have a program in Applied Mathematics, and we are at risk of losing some strong students who would be interested in UTM, but may not even apply because of this gap in our program offerings."

[4] https://www.mmf.utoronto.ca/ and https://www.sgs.utoronto.ca/programs/mathematical-finance/
[5] https://www.rotman.utoronto.ca/Degrees/PhD/Academics/MajorAreasofStudy/OperationsManagementandStatistics
[6] https://www.mie.utoronto.ca/industrial-engineering-research-area/
[7] https://web.cs.toronto.edu/research/numerical-analysis
[8] https://sgs.calendar.utoronto.ca/computer-science-applied-computing-mscac-applied-mathematics-concentration

4. Need and Demand

- Provide a brief description of the need and demand for the proposed stream focusing, as appropriate, on student interest, societal need, employment opportunities for prospective graduates, interest expressed by potential employers and accreditational bodies.

Mathematical models form the language of science. They allow us to extract meaningful information from the sea of data available to us, and to forecast outcomes. By way of example, one may think of the recent societal experience with the COVID pandemic. Epidemiological modelling (a field of mathematical sciences) became a key driver of medical decisions. It is not a coincidence that one of the leading sources of modelling in the province of Ontario was the Fields Institute for Research in Mathematical Sciences, which is arguably the centre of mathematical activity in the province. The crisis served to highlight how dependent we, as a society, are on mathematical modelling to make sense of the world around us.

Other examples abound: medical diagnostics, weather forecasting, climate analysis, banking and investing, searching the web, traffic management, and so on. All of these rely on sophisticated mathematical models. The Artificial Intelligence (AI) revolution we are witnessing now can be considered as essentially a mathematical analysis of data; analytical skills are needed for both designing and properly utilising AI tools.

This is reflected in the growth of jobs in these fields in North America: the US Bureau of Labor and Statistics currently projects that from 2021-2031 the job market for mathematicians and statisticians will grow by 31%, compared with the overall projected growth rate of 5%. They cite the need

> "…to inform business, healthcare, and policy decisions. The amount of digitally stored data will increase over the projections decade as people and companies continue to conduct business online and use social media, smartphones, and other mobile devices."

As a further reflection of this societal need, they quote the very high current median rate of pay of USD $112,110 per annum for a mathematician in the USA[9].

Companies across all industries seek out and eagerly hire graduates with the appropriate set of mathematical skills, which are common to all applied fields. This is the toolbox that we seek to develop in the proposed Stream 2. Crucially, the student population at UTM has long

[9] https://www.bls.gov/ooh/math/mathematicians-and-statisticians.htm

recognized the value of such education, which explains the current popularity of the Mathematical Sciences Minor with students in sciences and especially in Statistics. But the scope of a Minor program is not sufficient for a future expert career in mathematical sciences. This has long been recognized by our peer institutions, as described above. The introduction of Stream 2 within the Major in Mathematical Sciences will address this need at UTM as well.

Apart from improved employment prospects, Stream 2 will also open the door for the students to be admitted to one of the many graduate programs in, or cognate with, applied mathematics, including existing programs at U of T.
For instance:

- Operations Research is a branch of applied mathematics central to healthcare management among many other fields. Various masters and PhD programs housed at the Department of Mechanical & Industrial Engineering in the Faculty of Applied Science & Engineering would allow a student to focus on Operations Research.[10]
- The Master of Mathematical Finance program.[11]
- There is a Master of Public Health program offered through the Dalla Lana School of Public Health.[12]
- Graduate programs in Computer Science or Linguistics would allow students to do research in Numerical Analysis or Mathematical Linguistics.

All of these fields are mathematical in nature, and students who complete Stream 2 will be well placed to engage with studies in these graduate programs/areas of research and others like them.

To gauge the existing student interest in an Applied Mathematics stream, we have administered an email survey of students in MAT courses. 242 undergraduates responded. Of them, **89% would be either very interested (108 students, 45%) or somewhat interested (106 students, 44%) in a Major in Applied Mathematical Sciences**. Even though we had expected high interest, we were surprised by the level of students' enthusiasm for the idea. These numbers alone suggest strong and clear interest from students.

There is a particularly high interest among Statistics or Computer Science undergraduate students. In particular, 21% of the students who responded to the survey are either enrolled or are planning to enrol into a Statistics Major or Specialist, and 36% are either enrolled or are planning to enrol into a Computer Science Major or Specialist. As mentioned in the previous section, students in one of these Majors would currently be likely to take a Mathematics Minor, as well. Indeed, as of Winter 2023, 35% of the students in the Statistics Major POSt also had a Mathematics Minor POSt, (with a similar scenario for Computer Science).

[10] https://www.mie.utoronto.ca/industrial-engineering-research-area/
[11] https://www.mmf.utoronto.ca/
[12] https://www.dlsph.utoronto.ca/program/mph-epidemiology/

Many of these students would be better served by an Applied Mathematics Major stream if it were available. Being able to combine a program in Statistics or Computer Science with Stream 2 would be a career-enhancing opportunity for these students.

Looking outside of the Department of Mathematics and Computer Science, there are a number of programs that would make fantastic combinations with an Applied Mathematics stream. For instance, of the above survey responders, **20%** were either enrolled or planning to enrol into the Economics Major program. The pairing between Economics and the Mathematical Sciences Major has been traditionally popular with students. In the period from June 2020-2023, nearly **32% (26 out of 82)** of all students graduating with a Major in Mathematical Sciences and a Major in a subject outside of MCS had their second **Major in Economics**. These students, in particular, would benefit greatly in their future career prospects by taking the proposed Stream 2. This pairing would prepare students especially well for a career in Financial Mathematics. Furthermore, over the same period of time, **19.2% (138 out of 717)** of all students graduating with a Minor in Mathematical Sciences, had a **Major in Economics**. This is by far the largest cohort **(36% or 138 out of 384)** of all students with a Minor in Mathematical Sciences and a Major or Specialist program outside of the Department of Mathematical and Computational Sciences. It is natural to assume that many of these students would have elected to enter the proposed Stream 2 were it available, and as Stream 2 is introduced, the number of students choosing it over the Minor in Mathematical Sciences will rise. There is no concern of whether combining the two Majors would be feasible, since the track record of a double Major in Economics and Mathematical Sciences is already well established (and first year's requirements of the Mathematical Sciences Major – regardless of the stream completed – would not change).

Identical considerations apply to the next two most popular pairings of the Mathematical Sciences Major with a Major/Specialist outside of MCS: **Physics** (18.2% or 15 of 82 students with second Majors outside of MCS; 8.3% or 32 of 384 students with a Mathematical Sciences Minor combined with a Major or Specialist outside of MCS), and **CCIT** (8.5% or 7 of 82; and 10% or 38 of 384). The pairing of a Physics or CCIT Major with a Mathematical Sciences Major is well established, and there is a large group of students who would potentially be interested (and would greatly benefit in terms of their future career prospects) in switching to Stream 2 instead of a Mathematical Sciences Minor.

This data will help us focus our efforts in popularising Stream 2 towards the students in the relevant disciplines. It will also help guide the choice of topics for the "Topics in Applied Mathematics" courses (MAT386H5, MAT486H5). For instance, Introduction to Mathematical Finance has been previously offered by our department as a topics course and would be an excellent match.

In the table below we detail enrolments in the Mathematical Sciences – Major and the Mathematical Sciences – Minor programs from 2017 to 2021. We would anticipate that the enrolment numbers seen in the existing undifferentiated Mathematical Sciences – Major would be matched in the two new streams, with some increase as some students may choose Stream 2 over the Mathematical Sciences – Minor, going forward.

Program	Program Enrolment by Year				
	2017	2018	2019	2020	2021
Mathematical Sciences - Major	313	326	392	503	486
Mathematical Sciences- Minor	792	831	1075	1070	1021

Finally, it is possible that the current lack of an Applied Mathematics offering at UTM has been at least a factor in diverting enrollment from UofT towards other institutions. The new Stream 2 – Applied Mathematics may attract new students who otherwise would have attended a different institution.

5. Program Requirements

- Describe the requirements for the stream(s). (In doing so, please ensure that you explain how the requirements for the stream fit into the requirements for the program as a whole and relate to the requirements for the other existing streams.)

Proposed Program Completion and Program Enrolment Requirements for both Stream 1 and the proposed Stream 2 of the Mathematical Sciences Major are **listed in Appendix A**. The enrolment requirements are the same for both Stream 1 and Stream 2.

Students in both streams will take 2.5 credits in the same courses in their first year:

1. MAT102H5
2. MAT223H5 or MAT240H5
3. 1.0 FCE of Calculus:
   (MAT132H5 or MAT135H5 or MAT137H5 or MAT157H5) and (MAT134H5 or MAT136H5 or MAT139H5 or MAT159H5)
4. ISP100H5

This gives the students the same foundational set of courses and skills, regardless of the stream that they subsequently complete.

Students will apply directly to one or other of the streams in the Mathematical Sciences Major.

In second year, students in both streams must complete MAT244H5 (0.5 credit) and [(MAT232H5/MAT233H5) and MAT236H5] or MAT257Y5 (1.0 credit). In higher years, students in both streams must also complete MAT334H5 or MAT354H5 (0.5 credit). In total, 4.0 credits are shared in the completion requirements. There is further opportunity to complete additional shared courses, depending on the students' course choices to complete their second year requirements. In terms of upper year courses, students have many of the same electives available to them. The significant difference is that students in Stream 2 will be steered towards courses which develop key skills common to modern applications of mathematics. For example, MAT311H5 *Partial Differential Equations* and MAT322H5 *Mathematical Modelling in Biology*/MAT332H5 *Introduction to Nonlinear Dynamics and Chaos* are required rather than optional. In Stream 1 the emphasis will be on more generalist mathematical subjects to give students a broad exposure to topics within mathematical sciences themselves, for a future career development in mathematical sciences.

The program completion requirements for Stream 1 are the same as those for the current undifferentiated Mathematical Sciences Major program. Stream 1 also shares the same Program Learning Outcomes as the current undifferentiated Mathematical Sciences Major program. The majority of learning outcomes, aligned with the University Degree Level Expectations (UDLEs) are common to both streams (although some will be emphasised somewhat more in one stream than the other). The program learning outcomes and program design elements that are unique to Stream 2 are addressed in section 6, below.

As mentioned above, the required courses in the first year is identical for both streams. There is also the option for overlap in year two courses. A hypothetical future student who changes their mind regarding Stream 1 or 2 mid-enrollment (i.e. one who wishes to switch from one to the other) would have a clear path to leveraging courses taken aimed at one Stream in order to complete the other Stream. In other words, while the streams will have distinct elements, the existence of a common enrolment/completion core of 3.0 credits will allow students a measure of flexibility. Furthermore, students' ability to combine other Major programs with either stream of the Mathematical Sciences Major will be preserved, since the first-year offerings will be unchanged from the current program requirements.

6. Degree-Level Expectations (DLEs), Program Learning Outcomes and Program Structure

- Outline the learning outcomes as they relate to the proposed stream, underlining where these are similar to or different from those for existing fields/ concentrations. Indicate the means by which students will satisfy the relevant DLEs.

Preamble

The new Stream 1 will have the same learning outcomes in the existing undifferentiated Mathematical Sciences - Major program. The program learning outcomes and program design elements that are unique to Stream 2 are addressed in the table below.

All first-year students (who have not yet entered their POSts) will take MAT102H5 *Introduction to Mathematical Proofs*, plus a full year (1.0FCE) of Calculus. Despite the differences in the various Calculus courses, they prepare students to go up an escalating path of complexity and rigour. This gives students who might not be sure, for example, if they would like to take the Mathematical Sciences or Applied Mathematical Sciences stream of the Mathematical Sciences Major, the flexibility to decide later. And because the core set of skills is common and overlapping across the majority of our courses, it means that our program requirements can be fairly flexible in the upper years. In both programs, students have several 'baskets' of 300- and 400- level courses to choose from.

In addition to the program structure, the pedagogical approaches employed in the program also support students' skill development. Students in our lower-year courses will experience various forms of active learning course design, often based on a "flipped" or "semi-flipped" model. In these courses, students will typically complete some readings and other work, such as a pre-class quiz on said readings, before attending the first class where that material is discussed. These quizzes are no- or low-stakes, but occur each week, and put the first exposure to the material in the hands of students, and so help them increase their independence and learn and hone their (mathematical) reading skills. In some of these courses students will additionally receive guidance from their instructors on how best to read mathematics (because it is a very different activity from reading a novel, for example!)

As they move into upper-level courses, there will still be active learning designs in many of the courses, but they also will apply their skills from first and second year courses to work in a more independent, or at least, less-scaffolded way. Additionally, they will start to encounter courses which employ Inquiry-Based Learning (IBL), and other course assessments targeted at oral and written communication, as outlined below. In an IBL class, students are asked to present their solutions to problem sets in front of the class, reacting to and incorporating feedback, and answering questions from students and the instructor (this is a core feature of an "IBL" class.) Finally, in courses such as MAT315H5 *Introduction to Number Theory*, MAT382H5 *Mathematics for Teachers* and MAT392H5 *Ideas of Mathematics*, students will be assessed on writing assignments that are different from the core type (predominantly short-answer questions) described below and used in the majority of MAT courses, and instead will submit short essays or reports on particular mathematical topics, often that they research outside of class. More details of the various ways that the PLOs are 'satisfied' are below.

List of Program Learning Outcomes

Note: "PLO X.Y (**M**)" denotes a PLO that applies only to Stream 1: Mathematics Sciences Stream of the Major, while "PLO X.Y (**AM**)" is a PLO that applies only to Stream 2: Applied Mathematics Stream. All other PLOs apply to both streams in the Mathematical Sciences - Major.

PLO 1.1 Mathematical breadth, applicability, and multiple perspectives.
Students will be able to
PLO1.1a: Describe and elaborate on the mathematical concepts, tools, and definitions.
PLO1.1b: View their properties from multiple perspectives and translate between them.
PLO1.1c: Connect and synthesise concepts across various areas of mathematics and cognate disciplines.

PLO 2.1 Reading mathematics.
Students will be able to
PLO 2.1a: Effectively read a mathematical text, understanding what methodologies and tools are used.
PLO 2.1b: Validate computations and proofs.

PLO 2.2. Problem solving, proofs, and writing mathematics.
Students will be able to
PLO 2.2a: Identify, critique and explain the challenges in solving a mathematical problem or proof and relevant mathematical tools and ideas.
PLO 2.2b: Structure, plan and solve a problem through a conscious, deliberate, and iterative process.
PLO 2.2c: Write a solution or a proof clearly and concisely, using expert-like phrasing and structure.

PLO 2.3(M) Applying computers to understand and investigate mathematical structures.
Students will be able to
PLO 2.3a (M): Apply computers to aid visualisation and understanding of mathematical structures and to investigate and aid in the solution of mathematical problems.

PLO 2.3(AM) Applying computers to understand, investigate, and validate mathematical models, and presenting the results.
Students will be able to
PLO 2.3a (AM): Apply computers to aid visualisation and understanding of mathematical models.
PLO 2.3b (AM): Use computer experiments to investigate applied problems and to carry out numerical simulations of mathematical representations of natural processes.
PLO 2.3c (AM)Formulate possible solutions and describe modeling outcomes and mathematical tools involved in a deliberate, clear, and concise manner.

PLO 3.1(M) Mathematical inquiry and discovery.

Students will be able to
PLO 3.1a (M): Formulate questions about a mathematical subject and investigate.
PLO 3.1b (M): Develop and implement mathematical tools.

PLO 3.1(AM) Mathematical modeling and validation.
Students will be able to
PLO 3.1a (AM): Formulate mathematical models.
PLO 3.1b (AM): Validate mathematical models using both theoretical and numerical tools.
PLO 3.1c (AM): Explore the predictions arrived at through mathematical modeling.

PLO 4.1. Communication with peers/experts and with non-experts.
Students will be able to
PLO 4.1a: Collaboration within and across disciplines. Communicate effectively, both orally and in writing, with an audience of classmates, experts, non-experts, or peers from other disciplines, using clear and precise language.
PLO 4.1b: Participate effectively within a team to solve mathematical and/or multidisciplinary problems.

PLO 5.1(M) Limits of Mathematics.
Students will be able to
PLO 5.1a (M): Identify, describe, and discuss problems and topics in mathematics that lie at the edges of mathematical knowledge.

PLO 5.1(AM) Constraints of mathematical modeling.
Students will be able to
PLO 5.1a (AM): Identify the limitations of mathematical modeling and that mathematical tools themselves as well as their computer implementations have constraints which limit the extent of knowledge.

PLO 6.1. Independence, persistence, and productive mindset.
Students will be able to
PLO 6.1a: Independently investigate an unfamiliar area or piece of mathematics.
PLO 6.1b: Demonstrate persistence in problem solving, and a productive mindset about learning.

Degree Level Expectations	**Program Level Outcomes**	**How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes**	**Assessment of Teaching & Learning**
Depth and Breadth of Knowledge **Breadth of Knowledge:** Goals: In the course of their studies, students will gain an awareness and appreciation of the variety of modes of thinking, methods of inquiry and analysis, and ways of understanding the world that underpin different intellectual fields. Students will engage in critical thinking and analytical skills – including with respect to equity, diversity, and inclusion – through courses within and beyond their core field(s) of study, across	PLO 1.1 Mathematical breadth, applicability, and multiple perspectives. Students will be able to **PLO1.1a**: Describe and elaborate on the mathematical concepts, tools, and definitions. **PLO1.1b**: View their properties from multiple perspectives and translate between them. **PLO1.1c**: Connect and synthesise concepts across various areas of mathematics and cognate disciplines. Rationale: The spirit of this PLO is that students should leave the program with a broad level of experience in both understanding and applying	This PLO will be targeted, at least implicitly, by essentially every MAT course in our curriculum. Students will often be in situations in core assessments in their courses where knowledge from previous courses will be relied upon. Students will also regularly be asked to make translations between algebraic, geometric and other theoretical perspectives while solving problems and proving results, in more and more sophisticated ways throughout the curriculum. (PLO1.1a and PLO1.1b). For example, students will leverage concepts from multiple areas in courses such as MAT302H5 *Introduction to Algebraic Cryptography*, MAT311H5 *Partial Differential Equations*, MAT322H5 *Mathematical Modelling in Biology*, MAT332H5 *Introduction to Nonlinear Dynamics and Chaos*, MAT334H5 *Complex Variables*, and MAT406H5 *Mathematical Introduction to Game Theory*. In several upper-level courses, like	While assessment of this PLO will often happen in traditional exams and assignments, students will also be assessed using other means during their program, for example via a short paper (e.g. in MAT315H5 *Introduction to Number Theory*), a presentation (e.g. MAT392H5 *Ideas of Mathematics*, MAT309H5 *Introduction to Mathematical Logic*, MAT405H5 *Introduction to Topology*, MAT337H5 *Introduction to Real Analysis*, MAT247H5 *Algebra II*, MAT257Y5) or a portfolio (e.g. MAT344H5 *Introduction to Combinatorics*, MAT257Y5)

Degree Level Expectations	**Program Level Outcomes**	**How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes**	**Assessment of Teaching & Learning**
the humanities, the social and behavioural sciences, and the natural sciences. Depth of Knowledge: Students will attain depth of knowledge in their core field(s) of study through a progression of introductory, core, and specialized courses.	mathematical tools, computations, proofs, and theorems. They should recognize important mathematical concepts, and be able to draw connections between these concepts across fields. They should be familiar with unlocking a solution to a mathematical problem through recasting it from one perspective to another. Notes: Common to both streams. More emphasis on connections and applications within mathematical sciences (mathematics, statistics, computer science) in the Mathematical Sciences stream. More emphasis on connections and applications with other scientific disciplines in the Applied Mathematical Sciences stream.	MAT392H5, students will be required to pursue information from outside of the course materials and relate and synthesize it with what they've already learned. (PLO1.1c)	

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
Knowledge of Methodologies Goals: Students will have knowledge of and experience with different methodologies and approaches relevant to their core field(s) of study.	**PLO 2.1 Reading mathematics.** Students will be able to PLO 2.1a: Effectively read a mathematical text, understanding what methodologies and tools are used. PLO2.1b: Validate computations and proofs. Rationale: Reading mathematics is its own specialized skill. Here we aim to have students emulate the process that most mathematicians do naturally. For example, when reading a definition, a mathematician will immediately create examples and counterexamples. Experts will explore definitions using special cases and pathological or unexpected examples. When	Students will often be asked to read short sections of a course textbook each week for most of their MAT courses. However, since experts use many techniques to properly read a mathematics text (as mentioned above), we try to build a solid foundation in these skills in students' 100- and 200-level MAT courses. For example, MAT102H5 *Introduction to Mathematical Proofs*, MAT202H5 *Introduction to Discrete Mathematics*, and MAT223H5 *Linear Algebra* I all have explicit targeting of this skill embedded in the course work, wherein students are taught explicitly how best to approach reading a math text. This can include videos, handouts and 'read-alongs' where the instructor demonstrates or role-plays a student encountering actual course content for the first time, and the kinds of moves they could make to get the most from their reading.	While assessment of this PLO will often happen in traditional exams and assignments, students will also be assessed using other means. For example, as mentioned in column 3, students will be presented in tests and assignments, with unfamiliar mathematics, and will be assessed on their ability to decipher the work using the skills that make up "reading mathematics". Assessment will be changed as the student progresses to reading and interpreting more complex work, and examples found in historical sources. Independence on the students' part in interpretating what they read will be a key element in assessment in higher years.

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
	reading a theorem, experts recognize the hypothes(es) versus the result(s), they look for how and where hypotheses are used in a proof, among many other expert techniques. The majority of an undergraduate's mathematical experience will be reading the proofs and problem solutions of others. Therefore, it is essential that a student have the ability to verify them. The ability to read a proof or a problem solution and recognize a core idea is an essential skill for any mathematician: it provides insight into the mathematics itself, provides tools for obtaining other results, and is how a mathematician internalizes a result. Note:	Furthermore, these courses in various ways not only teach this skill but find novel ways to assess it, for instance by including a short piece of unfamiliar mathematics (like a new definition or example related to course material) for students to read and interpret and then work with on an assignment or test. Additionally, since many of our other 100- and 200- courses (e.g. all our Calculus courses) are flipped or partially-flipped, students will be working on this skill in basically all their early MAT courses. (PLO2.1a) Finally, while essentially all upper-year MAT courses assume that students are putting their foundation in this skill to work and improving it by reading course content on their own before and after class, some courses provide further explicit opportunities to advance this skill. For example, all	

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
	Common to both streams, this is an essential skill for any mathematically-literate graduate.	of our MAT courses taught with "Inquiry-Based Learning" (IBL) include an explicit requirement for students to read and make sense of course readings as a core activity (PLO2.1b) (e.g. MAT247H5 *Algebra II*, MAT337H5 *Introduction to Real Analysis*, MAT309H5 *Introduction to Mathematical Logic*, MAT405H5 *Introduction to Topology*). In a course like MAT392H5 *Ideas of Mathematics*, or in an independent study course, students will also learn to read work from current or historical mathematics publications (as compared to a textbook.) (PLO2.1a) Part of this ability to read mathematics will involve the related ability to validate the computations and proofs encountered through mathematical reading. This is a core skill that follows the ability to read and interpret mathematics (PLO 2.1b)	

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
	PLO 2.2. Problem solving, proofs, and writing mathematics. **Students will be able to** PLO 2.2a: Identify, critique and explain the challenges in solving a mathematical problem or proof and and relevant mathematical tools and ideas. PLO 2.2b: Structure, plan and solve a problem through a conscious, deliberate, and iterative process. PLO 2.2c: Write a solution or a proof clearly and concisely, using expert-like phrasing and structure. Rationale: Just as it is an essential skill to be able to read and validate mathematics, it is equally important to be able to create	Learning to validate one's own and others' mathematics is improved mostly implicitly through assignments and exams, and the reading tasks outlined in the previous PLO above, as well as through the frequent presentation and study of examples of calculations and proofs in class. However, many courses, especially lower-level ones, will explicitly target this skill by, for example, providing incomplete or incorrect proofs and solutions, and asking students to critique or complete them (PLO2.2a). Students could also be asked to give peer feedback, whether for marks or not (PLO2.2a), e.g. in MAT223H5 *Linear Algebra* I, where students swap proposed solutions to homework questions, give feedback to each other, then reflect and revise their work before submission (PLO2.2b, PLO2.2c). Students get detailed feedback, not just on the correctness of their work on	Assignments and exams will be used to assess these learning outcomes. In addition, students will be assessed through the presentation of their results in class. Students will also be assessed through exercises involving peer critique and feedback. Proof writing assessment will focus not only on the correctness of the proof, but also the manner in which it is written. This assessment quality will aid in the development of more refined abilities in this area.

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
	and write mathematical solutions and proofs in an understandable and verifiable manner. Problem solving is a core mathematical activity, and students should hone and mature their ability to recognize a well-crafted solution to a problem, and to find errors in poorly-crafted solutions. Having robust intuitions in a range of content areas about whether one approach or another is likely to be successful for solving a particular problem is an important sign of "mathematical maturity", as is an ability to re-use common problem-solving patterns and avoid common pitfalls.	assignments and exams, but actionable feedback to improve their writing in a range of courses. (PLO2.2b, PLO2.2c) In fact, specific writing-focused instruction and feedback have been incorporated into a range of MAT courses, in some cases with support from WDI (Writing Development Initiative) grants. MAT102H5 *Introduction to Mathematical Proofs*, in particular, is a whole course targeted at this broad PLO, building a foundation for students as they proceed to upper-level MAT courses. Courses at the 100/200 level with a focus on the proving and proof-writing side of this PLO include MAT102H5 *Introduction to Mathematical Proofs*, MAT157H5 *Analysis I*/MAT159H5 *Analysis II*, MAT202H5 *Introduction to Discrete Mathematics*, MAT224H5 *Linear Algebra II*, MAT240H5 *Algebra I*, MAT247H5 *Algebra II* and MAT257Y5. And courses with a focus on the problem-solving and problem	

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
		solution writing side include MAT13XH5 (Calculus courses), MAT232H5 *Calculus of Several Variables*, MAT236H5 *Vector Calculus*, MAT244H5 *Differential Equations I*, and MAT264H5. (PLO2.2c)	
	PLO 2.3(M) Applying computers to understand and investigate mathematical structures. Students will be able to PLO 2.3a (M): Apply computers to aid visualisation and understanding of mathematical structures and to investigate and aid in the solution of mathematical problems. Rationale: Computers are important and effective tools in the construction, exploration and solution of mathematical problems. Students should be able to use computational tools	This will be a relatively low-emphasis skill in the overall curriculum for the Mathematical Sciences - Major program (Stream 1 – Mathematics). However, there are required courses that teach this skill specifically, such as MAT244H5 and MAT236H5. Other courses, such as MAT402H5 *Classical Geometries* typically involve some amount of work with computers to investigate course content. (PLO2.3a) Additionally, in many courses throughout the curriculum, students will work with computational tools of various types (e.g. computer algebra systems, graphing and visualisation software) in and out of class to	Students will be assessed in their use of computers in select courses (described in the previous column) through assignments.

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
	to aid them in studying mathematics.	better understand course material, but will not specifically be assessed on this task (e.g. in our Calculus courses, and MAT202H5 *Introduction to Discrete Mathematics*, MAT302H5 *Introduction to Algebraic Cryptography*, MAT305H5 *Elementary Lie Theory*, MAT311H5 *Partial Differential Equations*, MAT315H5 *Introduction to Number Theory* and MAT406H5 *Mathematical Introduction to Game Theory*). We plan to increase the amount to which our offerings more explicitly target this skill; in particular, we plan to augment MAT305H5, MAT311H5 and MAT332H5 *Introduction to Nonlinear Dynamics and Chaos* (the Mathematics Major requires one of these three courses) with significant attention to development of this skill, in order to more heavily build on what is	

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
		done in MAT244H5 and MAT236H5.	
	PLO 2.3(AM) Applying computers to understand, investigate, and validate mathematical models, and presenting the results. Students will be able to PLO 2.3a (AM): Apply computers to aid visualisation and understanding of mathematical models. PLO 2.3b (AM): Use computer experiments to investigate applied problems and to carry out numerical simulations of mathematical representations of natural processes. PLO 2.3c (AM): Formulate possible solutions and describe modeling outcomes and mathematical tools involved in a deliberate, clear, and concise manner. Rationale:	Students will lay a foundation for this skill in their Calculus courses and then build on that in MAT244H5 *Differential Equations I*, where they will be introduced to the computational side of modeling with differential equations (PLO2.3a (AM)). But it is the new MAT264 course that will allow students to level-up their modeling skills, with its introduction of a variety of applications, tools, and computational approaches. Beyond this, students will then be able to take MAT322H5 *Mathematical Modelling in Biology* (a course specifically about modeling as it applies in Biology), as well as a variety of our planned upper-year Applied Mathematics topics courses. (PLO2.3b (AM), PLO2.3c (AM))	Students will be assessed in their use of computers in select courses (described in the previous column) through assignments.

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
	Numerical simulation of mathematical models is a core skill of an applied mathematician. Students should be able to use the available computational tools and to write basic code. Their future professional success may depend on being able to carry out a successful mathematical modeling experiment and to be able to explain the results.		
Application of Knowledge Application of Knowledge Goals: Students will be able to frame relevant questions for further inquiry within or beyond the core field(s) of study. They will be able to identify and apply the appropriate tools with	**PLO 3.1(M) Mathematical inquiry and discovery.** Students will be able to PLO 3.1a (M): Formulate questions about a mathematical subject and investigate. PLO 3.1b (M): Develop and implement mathematical tools.	Students get a strong introduction to these skills in the many flipped and partially-flipped lower level MAT courses. These are built on through a range of Inquiry-Based Learning courses, like MAT247H5 *Algebra II*, MAT257Y5, MAT309H5 *Introduction to Mathematical Logic*, MAT337H5 *Introduction to Real Analysis*, and MAT405H5 *Introduction to Topology*, as well as other courses (e.g. MAT202H5	Students will be assessed through tests, assignments, exercises, and presentations. Students will also have their expressions of understanding assessed in other ways, such as the portfolio creation and maintenance mentioned previously.

Degree Level Expectations	**Program Level Outcomes**	**How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes**	**Assessment of Teaching & Learning**
which they can address such questions effectively. This includes a knowledge of how historical and present discrimination (including, but not limited to, discrimination on the basis of race, religion, sexuality, gender, and ability) affect these questions, problems, and solutions.	**Rationale:** Formulating questions and hypotheses and investigating them is another core mathematical activity, and a typical path to new discoveries. Students should experience one of the most thrilling parts of mathematics - inquiry and discovery. An appropriate selection of mathematical tools and techniques is key to a successful mathematical exploration. Students should be able to formulate strategies and develop tools for their implementation.	*Introduction to Discrete Mathematics* and MAT344H5 *Introduction to Combinatorics*) that explicitly require students to develop both of these skills (PLO3.1a (M) and PLO3.1b (M)). This happens, for example, by instructors asking students to investigate an open-ended mathematics problem (e.g. 'what would be true if we changed this part of the statement of this theorem?' or 'can this be generalised to...?'), rather than something more typical that would have a single correct answer ('compute the derivative of this function'). Students in MAT344H5 are asked to build a portfolio that includes some mathematics related to, but beyond the course content that they've investigated, and students can take reading courses and various topics courses in upper-years that leave students space to	

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
		investigate topics on their own or in groups.	
	PLO 3.1(AM) Mathematical modeling and validation. Students will be able to PLO 3.1a (AM): Formulate mathematical models. PLO 3.1b (AM): Validate mathematical models using both theoretical and numerical tools. PLO 3.1c (AM): Explore the predictions arrived at through mathematical modeling. **Rationale:** The most exciting and rewarding part of Applied Mathematics is formulating a mathematical model which sheds light on a natural phenomenon and whose predictive power can be	As mentioned above, MAT264H5 is a course that is focused on modeling. Students will spend much of the time in that course developing these important skills for the Applied Mathematics stream (PLO3.1a,b,c (AM)). Again, this builds on the relatively light introduction to this topic they'll receive in their Calculus course (PLO3.1a (AM)), and then the more robust introduction in MAT244H5 *Differential Equations I*. MAT322H5 *Mathematical Modelling in Biology* builds on MAT264H5, and as we mentioned above, the planned Applied Mathematics topics courses will further advance these core skills. Also as mentioned above, MAT311H5 *Partial Differential Equations*, and to some extent MAT305H5 *Elementary Lie Theory* and MAT332H5 *Introduction to*	Students will be assessed through tests, assignments, exercises, and presentations.

Degree Level Expectations	**Program Level Outcomes**	**How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes**	**Assessment of Teaching & Learning**
	harnessed to understand and explore it. Students should develop a broad toolbox of mathematical modeling strategies and techniques. They should select the appropriate tools and to carry out an exploration using them. They should be able to understand the predictions of the model, to validate a model using both mathematical considerations and the available data, and to formulate the model's predictions and outcomes.	*Nonlinear Dynamics and Chaos* will have an increased emphasis on modeling in the future.	
Communication Skills Goals: Students will be able to effectively communicate and critically evaluate information, arguments, and analyses, using a range of modes of communication.	**PLO 4.1. Communication with peers/experts and with non-experts.** Students will be able to PLO 4.1a: Collaboration within and across disciplines. Communicate effectively, both orally and in writing, with an audience of classmates, experts, non-experts,	While the intensity and expectations on students around this PLO naturally scale up as the level of the MAT course increases, essentially all of our courses target this PLO on the writing side (PLO4.1a). This is because the core tasks on assignments and exams are specifically to communicate in the language of mathematics, the	As mentioned previously, students will be asked to communicate using various means. They will be assessed in part on their success in using these different forms of communication: for example via a short paper (e.g. in MAT315H5 Introduction to Number Theory), a presentation (e.g. MAT392H5 Ideas of Mathematics, MAT309H5

Degree Level Expectations	**Program Level Outcomes**	**How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes**	**Assessment of Teaching & Learning**
	or peers from other disciplines, using clear and precise language. PLO 4.1b: Participate effectively within a team to solve mathematical and/or multidisciplinary problems. Rationale: Mathematicians can be expected to communicate in a variety of modes (written vs oral) and to a range of audiences (lay vs expert). In many possible career paths, being able to communicate a key mathematical idea or argument to non-mathematicians is an important skill for a student to have. In this context, careful use of mathematical terminology is crucial (for instance, with a lay audience, the use of specialised jargon may be counter-	solutions and proofs that students formulate. On the oral side, some courses, such as all the IBL courses mentioned above (MAT247H5, MAT257H5, MAT309H5, MAT337H5, MAT405H5) as well as MAT382H5 *Mathematics for Teachers*, and MAT392H5 *Ideas of Mathematics* at the higher level will ask students to present their work in class in front of their peers. Peers are encouraged to ask questions and give constructive feedback, and instructors give advice and feedback on how to effectively present the content (PLO41.b). A few of our courses (e.g. MAT322H5 Mathematical Modelling in Biology, MAT332H5 Introduction to Nonlinear Dynamics and Chaos, MAT382H5 Mathematics for Teachers, MAT392H5, and MAT406H5	Introduction to Mathematical Logic, MAT405H5 Introduction to Topology, MAT337H5 Introduction to Real Analysis, MAT247H5 Algebra II, MAT257Y5) or a portfolio (e.g. MAT344H5 Introduction to Combinatorics, MAT257Y5)

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
	productive). Collaboration is a powerful and increasingly common way to solve mathematical problems and directly relies on effective communication. Being able to effectively work with others is a crucial "soft skill" for essentially all future careers. Note: Students in Stream 1: Mathematics will see more examples of communicating with peers from mathematical sciences (mathematics, computer science and statistics, etc), and students in Stream 2: Applied Mathematics will learn more about communicating with peers from non-mathematical sciences. But the objective of communicating effectively is both essential and common to both streams. In both streams, emphasis is placed more	Mathematical Introduction to Game Theory) additionally discuss how to communicate to non-experts through, for example, an assignment to create a short video that explains a topic, or a presentation to a (pretend) audience of high-school students (PLO4.1a,b).	

Degree Level Expectations	**Program Level Outcomes**	**How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes**	**Assessment of Teaching & Learning**
	heavily on communication with experts and peers.		
Awareness of Limits of Knowledge Goals: Students will acknowledge and appreciate the limits of their own knowledge. They will also gain an awareness of the uncertainty, ambiguity, and limits of our collective knowledge and how these might influence analyses and interpretations.	**PLO 5.1(M) Limits of Mathematics.** Students will be able to PLO 5.1a (M): Identify, describe, and discuss problems and topics in mathematics that lie at the edges of mathematical knowledge. Rationale: While students are not going to be able to go deeply enough into technical content to reach the forefront of mathematical research in a typical undergraduate degree, they should be aware that mathematics is active and evolving, and have some awareness of, and ability to describe where the boundaries between theorems, conjectures and the unknown (and unknowable) in mathematics lies.	Many of our courses introduce this to students naturally as mathematical theorems are introduced and discussed. Already in their first year, they will take MAT102H5 *Introduction to Mathematical Proofs*, which enculturates students to the way mathematicians make hypotheses (‘conjectures’) and explore what we can prove (or so far have proved). As students reach upper-level courses, instructors (either explicitly in class, or in readings, or assessments) start to put the various subject areas inside mathematics in relation to each other, and highlight areas of current research. Students in a course like MAT392H5 *Ideas of Mathematics* are asked to describe what they know about the boundaries of what is currently	Assignments and exams will be used to assess these learning outcomes. In addition, students will be assessed through the presentation of their results in class.

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
		known in a particular area. And in all of our IBL courses, as well as most other upper-level courses, students will be engaging in that research-like process of investigating mathematics and making new definitions, to explore the contours of what's true and false (PLO5.1a (M)). For example, in MAT344H5 *Introduction to Combinatorics*, there are often sharp boundaries between statements which are straightforward to find closed formulas to capture, and small adjustments for which these formulas shoot up in complexity or become infeasible - students experience the contours around which we have to be careful in mathematics.	
	PLO 5.1(AM) Constraints of mathematical modeling.	As with the previous PLO, many of our applied mathematics-focused	Assignments and exams will be used to assess these learning outcomes. In

Degree Level Expectations	**Program Level Outcomes**	**How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes**	**Assessment of Teaching & Learning**
	Students will be able to PLO 5.1a (AM): Identify the limitations of mathematical modeling and that mathematical tools themselves as well as their computer implementations have constraints which limit the extent of knowledge. Rationale: Students should know that mathematical modeling has innate constraints. For instance, multi-dimensional dynamic models are prone to chaotic behaviour which may be difficult to analyse and to simulate numerically. Constraints of this nature often require using simplistic "toy" models, which may not capture the full richness of the given natural phenomena. Awareness of modeling limitations is	courses introduce this to students naturally as new content is considered. Students will study the pros and cons of increased sophistication or complexity of a model in MAT264H5, and will be constantly encountering the boundaries of what is or is not feasible with a mathematical model in upper year courses, including applied mathematics topics courses, as well as courses like MAT322H5 *Mathematical Modelling in Biology* and MAT406H5 *Mathematical Introduction to Game Theory* (plus, as mentioned above, redesigned versions of MAT305H5 *Elementary Lie Theory*, MAT311H5 *Partial Differential Equations* and MAT332H5 *Introduction to Nonlinear Dynamics and Chaos*.)	addition, students will be assessed through the presentation of their results in class.

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
	essential, and their study is a major part of Applied Mathematics.		
Autonomy and Professional Capacity Goals: Students will acquire the skills, knowledge, and critical problem solving they need to become informed, ethical, inclusive, independent, and creative thinkers and decision-makers; gain an awareness and appreciation that knowledge and its applications are influenced by and contribute to society as a whole; and lay the foundation for learning as a life-long endeavour.	**PLO 6.1. Independence, persistence, and productive mindset.** Independently investigate an unfamiliar area or piece of mathematics. Demonstrate persistence in problem solving, and a productive mindset about learning. Rationale: Being able to effectively learn (at least some of) a new mathematical idea on one's own is extremely valuable to students who will use or interact with mathematics after their program is complete. Mathematics is an open-ended discipline; if one approach does not work, experts will backtrack	Students get a foundation for these skills in their lower-level courses, essentially all of which have some amount of independent work required, by virtue of being flipped or partially-flipped. As students reach the upper years, they will have been gradually introduced to more and more complex problems, which might require them to formulate a series of 'moves' in their work, including abandoning failed attempts and starting over. In this way, as reflected in the curriculum map, essentially all MAT courses play some part in developing these overlapping skills.	Assignments and exams will be used to assess these learning outcomes. In addition, students will be assessed through the presentation of their results in class.

Degree Level Expectations	Program Level Outcomes	How the Program Design/Structure of the Required Courses and Other Learning Activities Supports the Achievement of Program Learning Outcomes	Assessment of Teaching & Learning
	and try another approach (and we want students to emulate this practice). Failure is a necessary part of the process of mathematical problem solving and proving, and students should reflect on and learn from their failures as much as their successes. Note: Common to both streams.		

7. Assessment of Teaching and Learning

- Please describe the methods of evaluation for the various program requirements as they relate to the proposed stream.
- Describe how the methods for assessing student achievement are appropriate and effective relative to established program learning outcomes and DLEs. In other words, how will faculty be able to determine whether students have learned and can do what we expect them to by the end of the program?
- How will the program document and demonstrate the level of performance of students consistent with the University's DLEs?

The PLOs in the section above are organised under each University DLE and they include notes on how students will be ensured to satisfy each PLO. Each one includes some notes on assessment of that particular PLO.

Broadly, in MAT courses, the most common assessment types seen are summative in-person, written tests and final exams, with a mix (depending on the level and topic of a course) of computational/technical questions, and conceptual or proof-based short-answer questions. Most courses will also have some number of written take-home assignments, again with a mix of questions. These questions are most often short-answer questions of a variety of types on written or take-home/online assignments, or in-class tests and exams. Assessment of this type is very much standard practice in programs in our discipline, and by having the bulk of our assessments done this way gives students some measure of predictability and consistency and allows for a long period of incremental improvement over the course of their program. In general, as the year, and level of the course rises, the conceptual and/or computational complexity of the work students do increases.

That said, while we see these summative assessments as being a core feature of our courses and programs, there are a variety of other assessments that are used formatively in several of our courses, particularly those taken in the first and second year, to assess the PLOs. Indeed, many courses build off of this base assessment scheme, either by varying the format for how the assessments are delivered (e.g. a two-stage test, or an opportunity for students to revise and resubmit an assignment) and/or via a variety of types of additional or alternative assessments, many of which will be formative low/no-stakes, or include opportunities for instructor, peer, or automated feedback.

Furthermore, there are certain courses which specifically target, in a very explicit way, several of the PLOs above, via assessment types that are less common in our discipline, such as short article/essay assignments or other written reports, oral presentations, portfolios, group or independent projects, etc.

8. Consultation

- Describe the expected impact of what is being proposed on the nature and quality of the unit's/division's program of study and any impact on other units/divisions.
- Describe any consultation with the Deans of Faculties/divisions that will be implicated or affected by the creation of the proposed stream.

We have consulted extensively within the MCS Department, including the following:

- the Mathematics Curriculum sub-committee
- the MCS Curriculum Committee
- the Chair, the two Vice-chairs and the three Associate Chairs
- the Director of the Centre for Nonlinear Analysis and Modeling
- the Director of the Continuum Robotics Laboratory
- The UTQAP committee, which included a strong recommendation to proceed with the program in the UTQAP report.
- Various other MCS faculty and staff who might be involved in implementing and supporting the new program stream. All expressed support for the introduction of a stream in Mathematics and a stream in Applied Mathematics as a way to more clearly meet the needs of the students.

We have also consulted extensively with the Statistics Associate Chair, Prof. Luai Al Labadi, who is very positive and enthusiastic about the new Stream 2. He believes that Stream 2 will be a good complement to Applied Statistics programs.

Although there is not a direct impact, we have also consulted with the Associate Chair, Undergraduate Studies at the St George campus Department of Mathematics, Prof. Joe Repka. Prof. Repka was positive about new Stream 2 and did not foresee any issues.

The UTM Office of the Dean consulted with Offices of the Dean at the Faculty of Arts and Science (UTSG), the Faculty of Applied Science and Engineering (UTSG), and the University of Toronto Scarborough. The draft proposal was sent to the relevant units and stakeholders within these divisions for feedback.

There were no concerns from the University of Toronto Scarborough. The Faculty of Applied Sciences and Engineering (UTSG) had no questions and expressed no concerns. The Faculty of Arts & Science (UTSG) and received positive responses from the Department of Mathematics, the Department of Computer Science, and the Department of Statistical Sciences. The latter questioned the mention of Actuarial Sciences as an area that is to be addressed in the program, while finding no course in the completion requirements specific to the subject. This is in reference to Stream 2: Applied Mathematics. This observation is correct in that there is no course in Actuarial Science, but the intent is to have students learn mathematics skills that can be broadly applied to a wide range of disciplines, Actuarial Sciences included. While there is not a specific course on this topic, the curriculum will provide the students with the skills and knowledge to apply mathematical modeling and analysis in this field.

9. Resources

- Describe any resource implications of the change(s) including, but not limited to, faculty complement, space, libraries and enrolment/admissions.
- Please specify where this may impact significant enrolment agreements with the Faculty/Provost's office.
- Indicate if the major modification will affect any existing agreements with other institutions, or will require the creation of a new agreement to facilitate the major modification (e.g., Memorandum of Understanding, Memorandum of Agreement, etc.). Please consult with the Provost's office (vp.academicprograms@utoronto.ca) regarding any implications to existing or new agreements.

Both new streams are sustainable with current resources.

The new Stream 2 will not have significant resource implications outside of the Department of Mathematical and Computational Sciences. A student in Stream 2 will require very similar resources compared to a student in the existing undifferentiated Mathematical Sciences Major program, in terms of library resources, space, and faculty complement. There will be some additional administrative responsibilities for advising students, that will fall to existing administrative staff in the department.

In terms of teaching resources, the completion requirements of Stream 2 include a newly approved course: MAT264H5 - Introduction to Numerical Analysis. This course will be assigned to a permanent, core faculty member. We do anticipate that there could be an increased interest among faculty to offer upper year "topics" courses to engage with the applied mathematics central to Stream 2. Again, these will be taught by permanent core faculty members. If this is the case, requests for teaching support (sessional instructors) will be made to the Office of the Dean.

9.1 Faculty Complement

- Brief statement to provide evidence of the participation of a sufficient number and quality of faculty who will actively participate in the delivery of the program and the new stream.

As mentioned in the above section, we estimate an initial faculty investment of 1.0 FCE per year to start the stream, followed over time by a student/faculty ratio similar to the existing Major program. The current faculty members are qualified to deliver the courses in the new stream, as is. With increasing enrollment in the program over time, a new Teaching Stream faculty would be beneficial.

10. UTQAP Process

The UTQAP pathway is summarized in the table below.

Steps	Approving Body	Approval Date
Development/consultation within Unit	Ilia Binder Chair, Department of Mathematical & Computational Sciences	Dec. 19, 2023
Decanal & Provostial Sign-Off	Tracey Bowen Vice Dean, Teaching & Learning Marc Dryer Associate Dean, Academic Programs	Dec. 19, 2023
	VPAP sign-off	Dec. 18, 2023
Divisional Governance Approval	UTM Academic Affairs Committee	January 11, 2024
Submission to Provost's Office		March 2024
AP&P – reported annually		May 2024
Ontario Quality Council – reported annually		July 2024

APPENDIX A: Proposed Calendar Copy for Streams in the Mathematical Sciences – Major

A.1 Proposed Program Enrolment and Completion Requirements for Stream 1: Mathematics Stream

Enrolment Requirements:

Limited Enrolment — Enrolment in the Major program is limited to students with a minimum of 4.0 credits, including:

- MAT102H5 (minimum 60%);
- A minimum 60% grade in MAT134H5 or MAT136H5 or MAT139H5 or MAT137Y5 or MAT233H5 or a minimum 50% in MAT159H5 or MAT157Y5;
- ISP100H5; and
- A minimum cumulative grade point average (CGPA), to be determined annually.
- All students must complete 4.0 U of T credits before requesting this program. Courses with a grade of CR/NCR will not count as a part of the 4.0 credits required for program entry.

Completion Requirements:
8.0 credits are required.

First Year:

1. MAT102H5
2. [(MAT132H5 or MAT135H5 or MAT137H5 or MAT157H5) and (MAT134H5 or MAT136H5 or MAT139H5 or MAT159H5)] or (MAT137Y5 or MAT157Y5)
3. MAT223H5 or MAT240H5

Second Year:

1. MAT202H5 and MAT244H5
2. [(MAT232H5 or MAT233H5) and MAT236H5] or MAT257Y5
3. MAT224H5 or MAT247H5

Higher Years:

1. MAT301H5 and (MAT334H5 or MAT354H5)
2. MAT337H5 or MAT392H5 or MAT405H5
3. MAT305H5 or MAT311H5 or MAT332H5
4. MAT302H5 or MAT315H5 or MAT344H5
5. STA256H5 or CSC363H5 or 0.5 credit of MAT at the 300/400 level, except MAT322H5
6. 0.5 additional credits in MAT at the 400 level

NOTES:

1. MAT137H5 and MAT139H5 are recommended.
2. Students are strongly encouraged to familiarize themselves with the 100-level calculus pre-requisites to select the correct courses.
3. Students are strongly encouraged to enroll in MAT240H5 followed by MAT247H5.

A.2 Proposed Program Enrolment and Completion Requirements for Stream 2: Applied Mathematics Stream

Enrolment Requirements:

Limited Enrolment — Enrolment in the Major program is limited to students with a minimum of 4.0 credits, including:

- MAT102H5 (minimum 60%)
- A minimum 60% grade in MAT134H5 or MAT136H5 or MAT139H5 or MAT137Y5 or MAT233H5 or a minimum 50% in MAT159H5 or MAT157Y5;
- ISP100H5; and
- A minimum cumulative grade point average (CGPA), to be determined annually.
- All students must complete 4.0 U of T credits before requesting this program. Courses with a grade of CR/NCR will not count as a part of the 4.0 credits required for program entry.

Completion Requirements:
8.0 credits are required.

First year:

1. MAT102H5
2. [(MAT132H5 or MAT135H5 or MAT137H5 or MAT157H5) and (MAT134H5 or MAT136H5 or MAT139H5 or MAT159H5)] or (MAT137Y5 or MAT157Y5)
3. MAT223H5 or MAT240H5

Second year:

1. MAT244H5
2. STA256H5
3. [(MAT232H5 or MAT233H5) and MAT236H5] or MAT257Y5
4. STA260H5
5. MAT264H5

Higher years:

1. MAT311H5
2. MAT322H5 or MAT332H5
3. MAT334H5 or MAT354H5
4. MAT386H5 or MAT406H5 or MAT486H5 or STA312H5 or STA313H5 or STA348H5 or STA380H5
5. MAT332H5 or MAT322H5 or STA302H5 or STA312H5 or STA313H5 or STA348H5 or STA380H5 or CSC338H5
6. 0.5 additional credits in MAT at 300/400 level

NOTES:
MAT305H5 or MAT337H5 or MAT386H5 or MAT406H5 or MAT486H5 are recommended.